INVERSION OF MICROWAVE THERMOGRAPHIC DATA BY THE SINGULAR FUNCTION METHOD

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ABSTRACT

The potential of the singular function method for retrieving temperature profiles in biological structures from microwave radiometric data is examined.

A numerical simulation has been carried out in which the temperature distribution during hyperthermic treatments is reconstructed from radiometric measurements both at six and four frequencies. The results are compared and discussed in view of the practically relevant case of noise-corrupted data.

STATEMENT OF PROBLEM

The determination of the temperature distribution in the human body can be a valuable support in diagnostics and is a need during hyperthermic treatments. In the diagnostic application, since the temperature in neoplastic tissues is higher than that of the surrounding normal tissues, a localized increase of temperature must be detected. In hyperthermic applications, the temperature distribution in the tissues must be continuously monitored to ensure that the required heating rate in the tumor bulk is attained without concurrent damaging effects on the surrounding normal cells. The measurement of the emitted electromagnetic radiation in the microwave band offers a means for noninvasively monitoring the temperature distribution in the subcutaneous tissues, with some advantage with respect to other thermometric techniques using in situ point sensors.

This contribution refers to the interpretation of data in passive microwave sensing of nonhomogeneous biological structures. An inversion technique, through which the temperature information can be extracted from the radiometric data, is presented and discussed.

METHOD

For sake of simplicity, the subcutaneous region of tissues is modelled as a half-space where both the temperature and the physical properties vary only with the distance z from the body surface. By Fourier analyzing the electromagnetic emission of radiation from the thermal body in the lossless half--space $z < 0$, the plane wave spectrum of the brightness temperature $T_B(\omega,\underline{k})$ is obtained, where ω is the angular frequency of a monochromatic radiometric channel which measures the time-averaged electromagnetic flux (proportional to T_B), and $\underline{k}$ is the wave vector of the received radiation.

The brightness temperature of the biological structure can be written as

$$T_B(\omega,\underline{k}) = \int_{-L}^{0} T(z)\; W(\omega,\underline{k},z)\; dz \qquad (1)$$

where T is the physical temperature, W is a "weighting" function, and the lower limit of integration, $-L$, indicates the depth within the tissues beyond which the contribution to the brightness becomes negligible. W depends on the thickness and on the dielectric permittivity and conductivity of layers, as well as on frequency, polarization and angle of observation of the radiometer [1].

An inverse problem is defined by eq. (1), where the physical temperature $T(z)$ is the unknown in the Fredholm integral equation of first kind and a finite discrete set of measurements is given. This set can be generated by measuring the brightness temperature of the biological structure at various frequencies, polarizations and angles of observation. The effectiveness of the microwave radiometric technique depends on the degree to which significant features of the thermal pattern can be extracted from the brightness temperature measures. The form of the kernel of the Fredholm integral equation (1) denotes that this inverse problem is ill conditioned and that suitable inversion techniques must be used to optimally exploit the information content of the set of measurements. Several methods have been proposed and analyzed, mainly in the geophysical remote sensing context [2], [3], for solving eq. (1).

The singular function method [4] provides a means for improving the stability of the solution, giving, at the same time, useful information both on the number of significant measurements, and on the attainable resolution. Two bases $u(z)$ and $\underline{v}$ are introduced, the first in the function space of the weighting functions, and the latter in the n-dimensional space of

0149-645X/85/0000-0075 $01.00 © 1985 IEEE

brightness data. These two bases are defined through the shifted eigenvalue problem:

$$\int_{-L}^{0} W(p_\ell,z)\; u_k(z)\; dz = \lambda_k \; v_{k\ell} \tag{2}$$

$$\sum_{\ell=1}^{n} W(p_\ell,z)\; v_{k\ell} = \lambda_k \; u_k(z) \tag{3}$$

where the ℓ-th component of data vector $\underline{v}_k$, $v_{k\ell}$, corresponds to the set of measure parameters shortly denoted by p_ℓ. The unknown temperature $T(z)$, in turn, is obtained as a linear superposition of the singular functions $u_k(z)$ through the expression

$$T(z) = \sum_{k=1}^{n} \frac{\underline{v}_k \cdot \underline{T}_B}{\lambda_k} \, u_k(z) \tag{4}$$

containing the vector of the measured brightness temperatures $\underline{T}_B$. The discontinuities of the weighting functions at the borders of the various tissues induce corresponding discontinuities in the singular functions, which, consequently, are not appropriate to represent an essentially continuous function as the thermodynamic temperature. To circumvent this difficulty, the unknown temperature profile is sought in the space of the temperatures satisfying the appropriate heat-flow equation in the given structure when heat is supplied at a rate proportional to $W(p_\ell, z)$. When formulated in this space, the shifted eigenvalue problem (2), (3) originates a set of singular functions particularly suited to represent physical temperature distributions with a continuous behavior across the boundaries of the tissue layers [5].

RESULTS AND DISCUSSION

A numerical analysis has been conducted to assess the effectiveness and practicability of the singular function method to reconstruct the profile of temperature within a three-layer (skin-fat-muscle) region of tissues.

In practical situations, only few radiometric measurements are available, to cope with the complexity and cost of the radiometric system, as well as with the time for the acquisition and processing of data. The cases of both six and four measurements have been considered. The simulated data refer to observations in a direction perpendicular to the biological layers, conducted at different radiometric frequencies. The temperature distribution to be retrieved refers to the temperature profiles induced in the tissues by electromagnetic heating and surface cooling of variable duration. These profiles have been numerically calculated by using a nonlinear model of the thermal behavior of living tissues, which takes into account also local thermoregulating convective and conducting effects due to blood flow [6].

Results of the numerical simulation of the retrieval are presented in Figs. 1 a) to e), referring to the cases of four and six noiseless measurements in the band between 1.5 and 6.5 GHz with a radiometric bandwidth of 0.5 GHz. As it could be expected, the use of six radiometric channels improves the accuracy of the retrievals, specially in the deeper muscular tissue, which contributes less information because of the attenuation suffered by the emission. However, inspection of the singular values points out that both the last one and the fifth are one order of magnitude below the fourth one. This means that the noise which in real cases corrupts the measurements will be correspondingly amplified (see eq. (4)) producing instabilities in the retrievals. The choice of the number of radiometric channels, therefore, depends on the expected overall noise level of the radiometric system.

CONCLUSIONS

The numerical simulation that has been conducted on the basis of the singular function approach, indicates that temperature profiles in a monodimensional layered biological structure can be efficiently retrieved from noiseless microwave brightness data. The choice of the optimal radiometric channels is directly related to the errors in the measurements through the singular value spectrum of the radiative transfer operator. The presence of noise will eventually require robust retrieval/estimation algorithms, such as the Kalman filter, to improve the temperature reconstruction in actual situations [7].

REFERENCES

[1] F. Bardati, and D. Solimini, "Radiometric sensing of biological layered media", Radio Sci., vol. 18, pp. 1393-1401, 1983.

[2] A. Deepak, Ed., Inversion Methods in Atmospheric Remote Sounding, Academic Press, Inc., New York, 1977.

[3] S. Twomey, Introduction to the Mathematics of Inversion in Remote Sensing and Indirect Measurements, Elsevier, New York, 1977.

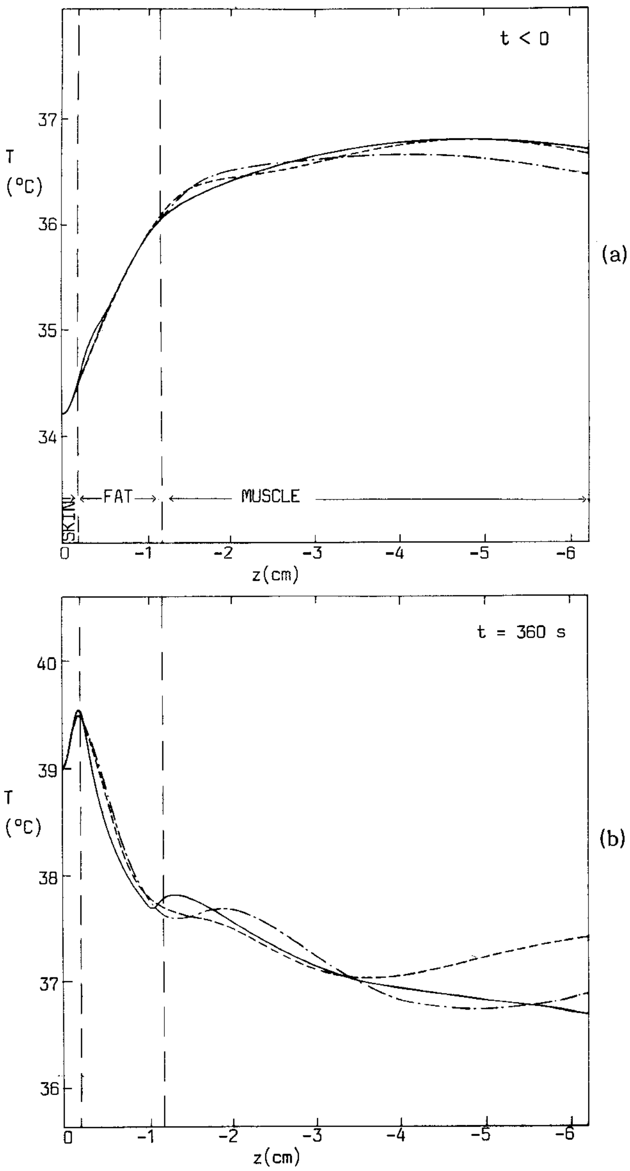
[4] M. Bertero, C. De Mol, and E.R. Pike, "Stability and resolution in electromagnetic inverse scattering", in Inverse Methods in Electromagnetic Imaging, W. - M. Boerner et al. (Eds.), D. Reidel Pub. Co., Boston, pp. 319-340, 1985.

[5] F. Bardati, M. Bertero, M. Mongiardo, and D. Solimini, "Biological temperature retrieval from microwave radiometric data", presented at the 1984 Rencontres Interdisciplinaires Problèmes Inverses, Montpellier, France.

[6] F. Bardati, and G. Gerosa, "Nonlinear thermal

behavior of electromagnetically irradiated multi-layered living tissues", presented at the URSI XXIst General Assembly, Open Symp. Interaction of Electromagnetic Fields with Biological Systems, Florence, Italy, 1984.

[7] M. Bertero, C. De Mol, and G.A. Viano, "The stability of inverse problems", in Inverse Scattering Problems, H.P. Baltes, Ed., Springer, New York, pp. 161-214, 1980.



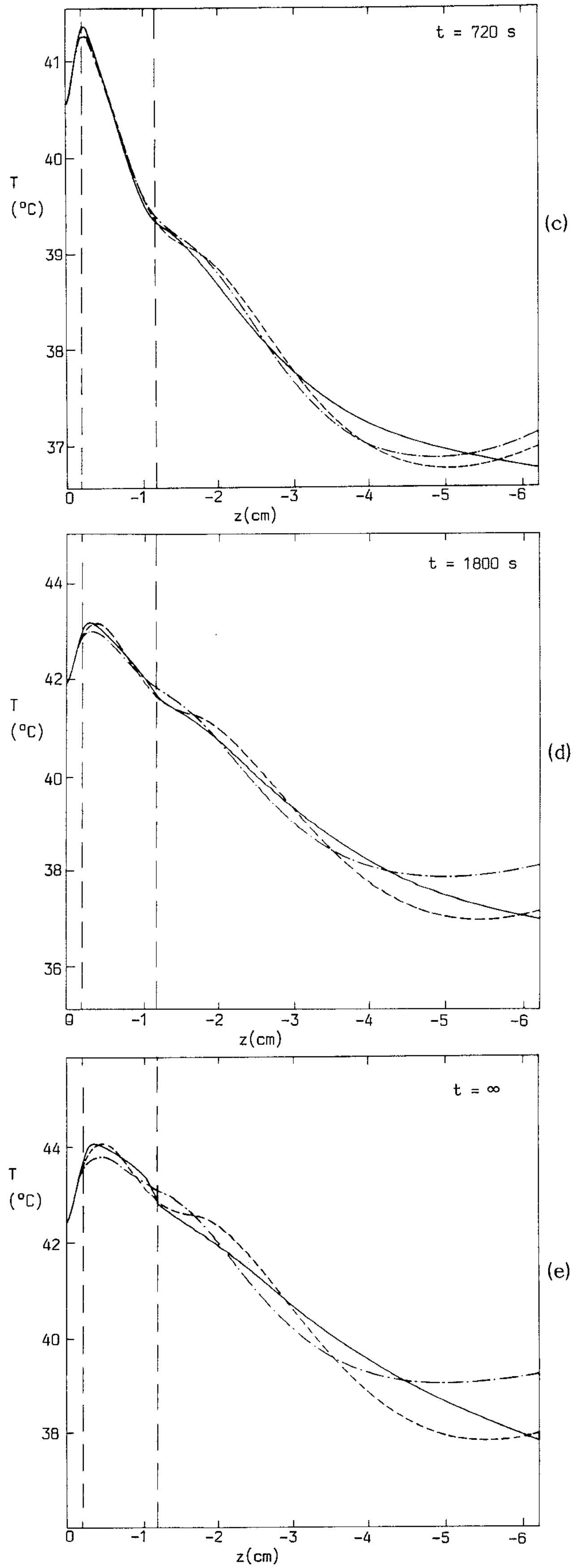


Fig. 1 Retrieval of temperature profiles in a skin-fat-muscle structure irradiated by an electromagnetic wave. The five diagrams from a) to e) refer to the times shown on the respective figures. The curves refer to the profile to be retrieved (continuous line), to the retrieval by using six equally spaced frequencies in the range 1.5-6.5 GHz (dashed line), and to the retrieval by using four frequencies (dotted line) in the same range.